HEURISTIC SOLUTION MODELS FOR THE SINGLE ITEM, UNCAPACITATED LOT-SIZING PROBLEMS

A THESIS
SUBMITTED TO THE DEPARTMENT OF MANAGEMENT
AND THE GRADUATE SCHOOL OF BUSINESS ADMINISTRATION
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF BUSINESS ADMINISTRATION

BY
DEMET ÇAPAN
JANUARY 1990

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January, 1990

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Business Administration.

Assist. Prof. Erdal Erel

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ABSTRACT

HEURISTIC SOLUTION MODELS FOR THE
SINGLE ITEM, UNCAPACITATED
LOT-SIZING PROBLEMS

Demet Çapan
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Single item, deterministic, periodic review, uncapacitated production/inventory models are especially important because of their applications in material requirements planning (MRP) systems. In this thesis, the relevant literature is reviewed and the performance of EOQ1, EOQ2, POQ, LUC, PPB, SM and GMC heuristic models are compared and discussed in the context of experimentation.

ÖZET

ENVANTER BÜYÜKLÜĞÜ TESBİT PROBLEMLERİNDE KULLANILABİLECEK TEK ÜRÜNLÜ, SINIRSIZ, SEZGİSEL ÇÖZÜM YÖNTEMLERİ

Demet Çapan
İş İdaresi Yüksek Lisans
Tez Yönetmeni : Yrd. Doç. Dr. Erdal Erel
Ocak 1990, 79 sayfa.

Tek ürünlü, deterministik, periodik inceleme ve sınırsız ürün/envanter modelleri malzeme ihtiyaç planlaması sistemlerinde yaygın uygulama alanına sahip olmaları açısından, özellikle önemlidirler. Bu tez çalışmasında, konuya ilişkin literatür özetle gözden geçirilmiş, EOQ1, EOQ2, POQ, LUC, PPB, SM and GMC isimli sezgisel yöntemlerin performansları deneylerle karşılaştırılmıştır.

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CHAPTER I

1.1. INTRODUCTION

The production/inventory process can be characterized by the flow of items into and out of storage points. The inflow of items is governed by the production or purchase acquisition, whereas the outflow of items is induced by demand associated with either customer orders or production orders. It may be impossible or uneconomical to balance exactly the inflow with the outflow, and consequently inventory is created at the storage points.

The production/inventory models which represent this process can be defined in terms of variables and their interactions. Some of the variables such as demand, cost and technology are uncontrollable, i.e., they are the parameters of the model. On the other hand, other variables such as production and inventory levels are controllable variables. Interactions can be represented in various forms such as, inventory balance equations and capacity constraints.

1.2. CLASSIFICATION OF PRODUCTION/INVENTORY MODELS

The production/inventory models can be classified into two groups Continuous review refers to the case where production (or purchase) decisions can be made at any point in time. In Periodic Review, decisions are made at discrete, usually equally-spaced points in time (i.e., the beginning of each day, month, etc.).

If the parameters of the model are known exactly, then the model is said to be a deterministic one, but if the parameters are random variables with known probability distributions, then the model is said to be stochastic.

Multi-item models are characterized by the fact that there exist cost, demand and resource interactions among the items. If there are no such interactions, then it becomes a single-item model.

If there exists a restriction on resources then the model is called capacitated, otherwise the model is considered as uncapacitated.

The models discussed in this study belongs to the class of single item, deterministic, uncapacitated, periodic review lot-sizing models. The choice is due to the wide, spread use of this class of models in MRP setting.

1.3. MODEL CONSTRUCTION

Following are the list of variables used in the model:

T = Planning horizon.

X_t = Production (purchase) quantity at period t, t=1,2,..., T

d_t = Demand at period t, t=1, 2, ..., T

I_t = Inventory level at the end of period t, t=1, 2 ,..., T

I_o = Inventory level at the beginning of the period.

c = Unit variable production (or purchase) cost.

h = Unit inventory carrying cost.

S = Set up (or ordering) cost.

The following is the mathematical programming formulation of the model:

$$\text{Min } Z = \sum_{t=1}^{T} \left[(S.y_t + c.X_t) + h.I_t \right]$$

Subject to

$$X_t + I_{t-1} - I_t = d_t \qquad t=1, 2, \ldots, T$$

$$X_t \le M\, y_t \qquad t=1, 2, \ldots, T$$

$$I_t \ge 0 \qquad t=1, 2, \ldots, T$$

$$X_t \ge 0 \;,\; y_t = \begin{cases} 1 & \text{If } X_t > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Where M is a large number.

1.2.1. The assumptions of the model:

The model has some important assumptions. These assumptions simplify the model and allow for mathematical manipulation and computational feasibility. They are listed below:

i) Demand is deterministic, i.e., demand quantities are known for all periods with certainty.

ii) The ordering, unit variable production and unit inventory carrying costs are deterministic and constant.

iii) No shortages are allowed; i.e., for any period, the demand can not be satisfied at later periods.

iv) Production (or purchase) decisions are made at the beginning of the periods.

v) The unit inventory carrying cost is a linear function of the inventory level. Also the unit variable production cost is a linear function of the production level; i.e., any other function is not allowed, because linearity feature of the objective function must be satisfied.

vi) Items are treated as independent items, i.e., there are no resource, demand or cost interactions among the items.

Solution of the problem above is usually computationally infeasible for a realistic T, since the number of constraints and variables are mostly affected by the size of the problem under consideration. On the other hand, the problem can be solved with a dynamic programming approach with much less computational requirements. Such a formulation was first given by Wagner and Whitin (12). Although Wagner and Whitin (WW) model gives optimal solution, it requires relatively high computational effort in MRP environment; i.e., the WW model searches T(T+1)/2 alternative solution procedures. For that reason, several lot-sizing heuristic models are proposed in the literature. Their computational requirements are relatively less but they do not ensure optimality. In this study, it is examined seven heuristic models and apply them to 35 test data(developed by Kaimann [5]) to find out which of these seven heuristic models most closely approximates the optimal solutions found by the WW.

1.3.2. IMPLICATIONS OF THE ASSUMPTIONS

The meaning and importance of the assumptions and the implications of relaxing them are briefly discussed below:

(i) If the demands were not deterministic, then it would necessitate the use of probabilistic distribution functions for expressing the demand set, in which case a linear model could not be used, nor could such a model be deterministic.

(ii) In the heuristic models, variables such as set-up costs, inventory carrying costs, production costs, etc. are assumed to be constants. Relaxing this assumption, would invalidate the use of heuristic models and would necessitate the use of an optimum finding algorithm such as WW model.

(iii) The assumption of "no shortages" : This assumption can be relaxed easily, because if it were to be relaxed then we would have to introduce another cost term into our objective function and make our evaluations accordingly, in which case the extra term is the product of number of shortages and unit shortages cost.

(iv) Production (purchase) decisions are made at the beginning of the periods. In other words, this is the "periodic review assumption".

(v) The unit inventory carrying cost is a linear function of the inventory level. The relaxation of this assumption would render our problem non-linear.

(vi) The assumption of " single-item ": Otherwise, the problem is multi-item lot-sizing problem. But, multi-item, uncapacitated problems are also solved for each item by using this model.

1.4. PURPOSE OF THE THESIS

"The single item, deterministic, periodic review and uncapacitated production/inventory model" is chosen as a subject for the thesis mainly because of applications in MRP systems, since the production planning environments are generally affected by the decisions to be made on MRP systems. This thesis is based on comparison of seven heuristic models to solve lot-sizing problems.

1.5. THESIS OUTLINE

In the introduction chapter, a classification of the production/inventory models is considered. Chapter I states variables which are used in the model considered and the mathematical programming formulation of the model is given. The assumptions of the model and their implications are also presented in Chapter I.

The analysis of the available heuristic models with their assumptions and other structural properties are presented in Chapter II.

In Chapter III, properties of data sets for computational comparison of heuristic models are described. Evaluation of the results and the comparison of the heuristic models are presented. The conclusions and recommendations are considered in Chapter IV.

CHAPTER II

HEURISTIC MODELS

In the literature, several heuristic models for determining lot sizes in single item, deterministic and periodic review models, have been outlined. The optimal solution to the problem can be obtained by the Wagner and Whitin model(12).

The W/W model is a dynamic programming approach which uses several theorems to simplify the computations. The algorithm proceeds in a forward direction to determine the minimum cost policy. Although it finds an optimal solution, heuristic models are generally used in practice since computational requirements are quite large. Such that, the computation time of WW algorithm increases explonantially relative to heuristic models´ computation times when the size of parameters increases. Thus, various heuristic models have been developed since 1968. These heuristic models are computationally more attractive, but they do not ensure optimality.

The following heuristic models are frequently referred to in the literature:

1-Economic Order Quantity (EOQ) (1,6,8,9)
2-Period Order Quantity (POQ) (9,10)
3-Least Unit Cost (LUC) (3,8)
4-Part Period Balancing (PPB) (2,7,8,11)
5-Silver and Meal Heuristic (SM) (9,10,11)
6-Groff Marginal Cost Algorithm (GMC) (4,11)

The first one is demand-rate oriented and the rest are discrete lot-sizing procedures. Since the discrete lot-sizing procedures generate order quantities which equal the sum of demands in an integral number of consecutive planning periods, they do not create "remnant" stock; i.e., quantities that would be carried in the inventory for a length of time without being sufficient to cover a future period's demands in full.

In some of these models, the order quantity is fixed while the ordering interval varies; EOQ belongs to this class. In POQ, on the other hand, the ordering interval is fixed and the order quantity varies. The rest, including LUC, PPB, SM and GMC allow both ordering interval and order quantity to vary. Thus, they have the capability of coping with the seasonal variability or lumpiness of the demand. For this reason, the last four heuristic models are widely used in practice.

In the literature, there are two inventory carrying cost criteria: " Average inventory carrying cost (AICC) criterion " and " End of period (EOP) criterion ". The basic difference between the two criteria can also be demonstrated in the following manner:

Let H(n) denote the inventory carrying cost for n periods using the end of period criterion. And let H′(n) denote the inventory carrying cost for n periods using the average inventory carrying cost criterion. It can be expressed as follows:

$$H'(n) = h \sum_{t=1}^{n} I_t \left[t - \frac{1}{2} \right]$$

$$H'(n) = h \sum_{t=1}^{n} I_t \; (t-1) + h \frac{1}{2} \sum_{t=1}^{n} I_t \qquad (2.1)$$

Whereas

$$H(n) = h \sum_{t=1}^{n} I_t \; (t-1)$$

As it can be observed, the end of period criterion differs from the average inventory carrying cost criterion by the second term of Eq. (2.1). This term has no effect on the optimal solution of W/W model since it would be added identically to all ordering alternatives for each period. Since optimal solution does not change with changing the inventory cost criterion, in all heuristic models end of period criterion is used.

In the rest of this chapter, the basic concepts of each of the seven heuristic models will be summarized; the solution procedures will be developed and stated and they will be applied to a simple set of demand data in lieu of an example.

2.1. ECONOMIC ORDER QUANTITY MODEL

This model is widely used because of its simplicity. The EOQ is based on the assumption that the demand is continuous and it is calculated from the formula (2.2).

Since the class of models considered in this study is periodic review(i.e., demand occurs at discrete points in time), the policy for determining the order point is therefore modified.

$$EOQ = \sqrt{\frac{2SD}{h}} \tag{2.2}$$

In all the examples of this chapter, data set given in Table 2.1. is used. The computations for ordering and carrying costs are also shown. The carrying cost is found by adding the ending inventory for all periods and then multiplying the sum by the carrying cost per unit time(6).

Table 2.1. Data Set for the Examples

period no	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10

S =TL 300.
h =TL 2. per unit per period

There are two variations of the EOQ heuristic model:EOQ1 and EOQ2.

2.1.1 ECONOMIC ORDER QUANTITY 1 MODEL

This heuristic model places an order whenever the quantity on hand is less than current demand. A check is made to see if EOQ equals or exceeds current demand. If so, the order quantity is the EOQ. If EOQ is less than the current demand, then the order quantity is increased to meet the current demand. In brief, production quantity is set equal to the maximum of EOQ or the difference between demand at current period and inventory level at previous period(6).

If $I_{t-1} - d_t \geq 0, \quad X_t = 0$

otherwise, $X_t = \max \left\{ EOQ \, , \, d_t - I_{t-1} \right\}$

stopping rule $I_{t-1} < d_t$

An example illustrating this heuristic model is given Table 2.2. by using EOQ1 procedure.

Table 2.2 EOQ1 Example

Period no :	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Production Quantity	575	0	0	0	0	0	0	575	0	0	0	0
Beginning Inventory	575	565	555	540	520	450	270	595	325	95	55	55
End Inv.	565	555	540	520	450	270	0	325	95	55	55	45

Total Ordering Cost = 2x300 =TL 600.

$$EOQ=\sqrt{\frac{2SD}{h}} \quad \sqrt{\frac{2x300x1105}{2}} = 575 \text{ units}$$

Total inventory carrying cost=TL 6990.

2.1.1.1 EOQ1 PROCEDURE

1) Initialization, $t = 1$, $I_o = 0$, $X_t = 0$, $\forall t$ go to $\boxed{2}$

2) $X_t = \max \left\{ EOQ \; , \; d_t - I_{t-1} \right\}$ go to $\boxed{3}$

3) Let, $I_t = \sum_{k=1}^{t} X_k - \sum_{k=1}^{t} d_k$ go to $\boxed{4}$

4) If $I_t > d_{t+1}$ then, go to $\boxed{5}$

else $t=t+1$ go to $\boxed{2}$

5) $t=t+1$

If $t < T$ then, go to $\boxed{3}$

else go to $\boxed{6}$

6) End.

2.1.2 ECONOMIC ORDER QUANTITY 2 MODEL

The EOQ2 model is almost the same as the EOQ1. The only difference is that the EOQ2 is a discrete lot sizing model. That is, the order quantity is equal to the cumulative sum of the demand of consecutive periods. But it is not necessarily equal to the EOQ. In this model, the lot-size is determined by comparing the EOQ with the cumulative demands at the consecutive periods. The one which is closer to the EOQ is chosen as the lot size (1). The stopping rule of this heuristic model is as follows:

$$EOQ < \sum_{t=1}^{k} d_t$$

When the rule above holds, the order quantity is taken to be $\sum_{t=1}^{k-1} d_t$ or $\sum_{t=1}^{k} d_t$, whichever is closer to the EOQ.

The following are the definition of symbols used in the EOQ2 procedure :

- p is the current period at which we are making decision.
- k is the period at which sum of cumulative demand from period p to k just exceeds EOQ.

let, $$A_{pk} = \sum_{t=p}^{k} d_t$$

$$B_p = \sum_{i=p}^{T} d_i$$

$$\text{If } d_p > EOQ \text{ then, } \quad X_p = d_p$$

$$\text{else If } B_p < EOQ \text{ then, } \quad X_p = B_p$$

$$\text{else If } | A_{pk} - EOQ | < | A_{pk-1} - EOQ | \quad X_p = \sum_{t=p}^{k} d_t$$

$$\text{else then, } \quad X_p = \sum_{t=p}^{k-1} d_t \ .$$

As shown in Table 2.3., for example the stopping rule holds for the first time until period 8. Then the size of first lot is taken to be 555. Since the cumulative demands of first five periods (555) is closer to the EOQ (575) than the cumulative demand of first eight period (825).

Table 2.3. EOQ2 Example

Period no	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Order Quantity	555							550				
Beginning Inv.	555	545	535	520	500	430	250	550	230	50	10	10
Ending Inv	545	535	520	500	430	250	0	230	50	10	10	0

Ordering Cost = 2(TL 300) =TL 600.

Inv. carrying cost =TL 6280

2.1.2.1 EOQ2 PROCEDURE

1) let, $p=1$, $k=1$, $X_t = 0$ $\forall t$

2) If $k > T$ then go to $\boxed{7}$

   else go to $\boxed{3}$

3) If $d_p > EOQ$ then, $X_p = d_p$ go to $\boxed{7}$

   else go to $\boxed{4}$

4) Find A_{pk}, B_p go to $\boxed{5}$

5) If $B_p < EOQ$ then, $X_p = B_p$ go to $\boxed{7}$

   else go to $\boxed{6}$

6) If $| A_{pk} - EOQ | < | A_{pk-1} - EOQ |$ then,

$$X_p = \sum_{t=p}^{k} d_t \quad ; \quad p = k+1$$

else $$X_p = \sum_{t=p}^{k-1} d_t \quad ; \quad p = k$$

go to $\boxed{2}$

7) End.

2.2. PERIOD ORDER QUANTITY MODEL

Period order quantity model is based on the principles of the classic EOQ(9)model. In POQ procedure the economic ordering interval (EOI) is computed rather than the economic order quantity. Once the EOI is computed, the lot-size is taken as the sum of demands during the economic ordering interval. This heuristic model is equivalent to the simple rule of ordering "x months´supply", but x is computed rather than determined exogenously.

$$\text{OPH (Orders per horizon)} = \frac{D_t}{EOQ} \qquad \text{which is}$$

$$D_t = \sum_{t=1}^{T} d_t$$

$$\text{EOI(economic order interval)} = \left[\!\left[\frac{T}{OPH} \right]\!\right]^*$$

$$k_t = \min \left\{ t+EOQ-1,\ T \right\}$$

$$Q_t = \sum_{i=t}^{k_t} d_i$$

$$X_t = \begin{cases} 0 & \text{If } (I_{t-1} > 0) \text{ or } (d_t = 0) \\ Q_t & \text{Otherwise} \end{cases}$$

$^*\left[\!\left[a \right]\!\right] = b$, b is the largest integer which is greater than or equal to a

Using the data given in Table 2.1., the EOI is computed as follows:

EOQ = 166

Number of period in a horizon = 12

Total demand in a horizon =1105 units

$$OPH = \frac{1105}{575} = 1.92 \cong 2 \quad \text{(order per a horizon)}$$

$$EOI = \frac{12}{2} = 6 \quad \text{(economic ordering interval)}$$

Application of the POQ heuristic to the sample problem is given in Table 2.4 by using Section 2.2.1.

Table 2.4. Period Order Quantity Example

Period no	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Order Quantity	305						800					
Beginning Inv.	305	295	285	270	250	180	800	550	280	50	10	10
Ending Inv.	295	285	270	250	180	0	550	280	50	10	10	0

Ordering Cost =2(TL 300.) =TL 600.

Inv. carr. Cost= 2*2180 =TL 4360.

2.2.1. POQ PROCEDURE

1) Initialization, $t=1$, $X_t=0\ \forall t$, Find EOQ, EOI

2) If $d_t=0$ then, $t=t+1$

   else compute k_t, Q_t go to $\boxed{3}$

3) Set $X_t=Q_t$ and $K=k_t$

4) Set $t=K$, $t=t+1$

5) If $t=T$ go to $\boxed{6}$

   else go to $\boxed{2}$

6) End

2.3. LEAST UNIT COST MODEL

Least Unit Cost model is based on the minimization of the "unit cost". In determining the lot-size the LUC model probes whether the lot-size should be equal to the first period's demand or whether it should be increased to cover the demand of the next period and/or the one after that, etc. The decision is based on the "unit cost" (i.e., set up plus inventory carrying cost per unit) computed for each of the successive order quantities. The one with the least unit cost is chosen to be the lot-size.

Derivation of the stopping rule of the LUC heuristic is as follows (3): UC(n) is defined as the unit cost of replenishment which covers n period's demand.

$$UC(n) = \frac{1}{\sum_{t=1}^{n} d_t} \left[S + h \sum_{t=1}^{n} d_t(t-1) \right] \qquad (2.3)$$

The basic idea of the LUC heuristic is to evaluate Eq.(2.3) for increasing values of n, until the following condition is satisfied:

$$UC(n+1) > UC(n) \qquad (2.4)$$

that is, until unit cost starts to increase.

Using Eq.(2.3) and Eq.(2.4), we can obtain

$$\frac{1}{\sum_{t=1}^{n+1} d_t} \left[S + h \sum_{t=1}^{n+1} d_t(t-1) \right] > \frac{1}{\sum_{t=1}^{n} d_t} \left[S + h \sum_{t=1}^{n} d_t(t-1) \right]$$

Last inequality can be expressed as:

$$\frac{\dfrac{S}{h} + \displaystyle\sum_{t=1}^{n+1} d_t(t-1)}{\dfrac{S}{h} + \displaystyle\sum_{t=1}^{n} d_t(t-1)} > \frac{\displaystyle\sum_{t=1}^{n+1} d_t}{\displaystyle\sum_{t=1}^{n} d_t} \qquad (2.5)$$

by defining a counter $F(n)$ as:

$$F(n) = F(n-1) + (n-1).d_n \qquad n = 2,3,...$$

where

$$F(1) = \frac{S}{h}$$ and

by substituting $F(n)$ into Eq.(2.5), we can obtain

$$\frac{F(n+1)}{F(n)} > \frac{\displaystyle\sum_{t=1}^{n+1} d_t}{\displaystyle\sum_{t=1}^{n} d_t} \qquad (2.6)$$

Finally,

$$n \sum_{t=1}^{n} d_t > F(n) \qquad (2.7)$$

The equation (2.7) is the stopping rule of the LUC heuristic.

In summary, the LUC heuristic determines the lot size by evaluating expression Eq.(2.7) for the increasing values of n and adding the future demands to the current lot until the stopping criterion is satisfied. The same procedure is repeated for the remaining periods. Application of this heuristic to the sample problem is given in Table 2.5 by using Section 2.3.1.

Table 2.5. Least Unit Cost Example

Period No.	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Order Quantity	125					180	250	270	230	50		
Beginning Inventory	125	115	105	90	70	180	250	270	230	50	10	10
Ending Inventory	115	105	90	70	0	0	0	0	0	10	10	0

Ordering Cost = 6(TL 300.) =TL 1800.

Inv. carr. cost =TL 800.

Total Cost =TL 2600.

2.3.1 LUC PROCEDURE

1) let, $p=1$, $F(0)=0$, $X_t=0 \quad \forall t$

2) $n=1$, Since, $F(1)=\frac{S}{h}$

   If $p=T$ then $X_t = d_t$ go to $\boxed{9}$

   else go to $\boxed{3}$

3) If $d_p \leq F(1)$ then, go to $\boxed{5}$

   else go to $\boxed{4}$

4) Set $X_p = d_p$ and $p = p+1$ go to $\boxed{2}$

5) If $p+n-1 = T$ then, $X_p = \sum_{t=p}^{T} d_t$ go to $\boxed{9}$

   else $n=n+1$ go to $\boxed{6}$

6) Compute $\text{cost} = n \sum_{t=p}^{p+n-1} d_t$, $F(n) = F(n-1) + (n-1)d_{p+n-1}$

7) If $\text{Cost} \leq F(n)$ go to $\boxed{5}$

   else go to $\boxed{8}$

8) Set, $X_p = \sum_{t=p}^{p+n-1} d_t$, $p = n+1$ go to $\boxed{2}$

9) End

2.4. PART PERIOD BALANCING MODEL

PPB heuristic has the objective of minimizing the sum of set up and inventory carrying cost. It attempts to reach this objective by trying to equate the total cost of ordering to the total cost of carrying inventory. This can be written as follows:

$$S \approx h \sum_{t=1}^{n} d_t \ (t-1)$$

dividing both sides by h, we obtain

$$\frac{S}{h} \approx \sum_{t=1}^{n} d_t \ (t-1) \qquad (2.8)$$

In the literature(7), S/h is called "The Derived Part Period Value" and some authors call it" The Economic Part Period (EPP)". It is the inventory quantity which, if carried in the inventory for one period, would result in a carrying cost equal to the set up cost. The RHS term, $\sum_{t=1}^{n} d_t \ (t-1)$, is known as "The Generated Part Period Value", or as some authors call it, "The Part Period Cost". It is the number of items held in the inventory for a certain period of time.

The PPB heuristic selects the order quantity such that The Part Period Cost (Generated Part Period Value) is close to the EPP (Derived Part Period Value). The stopping rule of this heuristic is as follows (2):

$$\frac{S}{h} < \sum_{t=1}^{n} d_t(t-1) \qquad (2.9)$$

that is, when the generated part period value is first greater than derived part period value, new order should be placed. To determine reorder period and the order quantity, the generated part period values of the current and the previous periods are compared with derived part period value. Reorder period is the successor of the period at which the derived and the generated part period values are the closest. Then, the order quantity equals the cumulative demands up to the preceding period This can be presented as follows:

When the stopping rule holds (i.e., $\frac{S}{h} < \sum_{t=1}^{n} d_t(t-1)$)

Let $DIF_n = \sum_{t=1}^{n} d_t(t-1) - \frac{S}{h}$

and

$$DIF_{n-1} = \frac{S}{h} - \sum_{t=1}^{n-1} d_t(t-1)$$

then, the first lot-size will be :

$$X_1 = \begin{cases} \sum_{t=1}^{n} d_t & \text{If } DIF_n < DIF_{n-1} \\ \sum_{t=1}^{n-1} d_t & \text{If } DIF_n \geq DIF_{n-1} \end{cases}$$

and the reorder period (or period at which new order is placed) will be :

$$j = \begin{cases} n+1 & \text{if } DIF_n < DIF_{n-1} \\ n & \text{if } DIF_n \geq DIF_{n-1} \end{cases}$$

To put the stopping rule in a form which is compatible with that of LUC, the following algebraic manipulation can be made:

$$\frac{2S}{h} < \frac{S}{h} + \sum_{t=1}^{n} d_t \,(t-1) \qquad (2.10)$$

by defining a counter F(n) as :

$$F(n) = F(n-1) + (n-1)d_n \qquad n=2,3,...$$

where

$$F(1) = \frac{S}{h}.$$

and substituting these into Eq.(2.10), we can obtain the stopping rule of PPB heuristic in the following form:

$$\frac{2S}{h} < F(n) \qquad (2.11)$$

Application of the PPB heuristic to the sample problem is illustrated in Table 2.6 by using Section 2.4.1.

Table 2.6. Part Period Balancing Example

Period No.	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Order Quantity	55				250		520		280			
Beginning Inventory	55	45	35	20	250	180	520	270	280	50	10	10
Ending Inventory	45	35	20	0	180	0	270	0	50	10	10	0

Ordering Cost = 4(TL 300.) =TL 1200.

Inv.carr. Cost = 2(TL 620.) =TL 1240.

Total Cost =TL 2440.

2.4.1. PPB PROCEDURE

1) Let, j=1, t=1, $X_t = 0 \quad \forall t$, $F(1)=\frac{S}{h}$, $CST= \frac{2S}{h}$

2) If t=T or j=T then, $X_j = \sum_{p=j}^{T} d_p$ go to $\boxed{7}$

else t=t+1 go to $\boxed{3}$

3) Set k=t-j+1

$F(k) = F(k-1) + (k-1)d_t$

4) If $F(k) \leq CST$ then, go to $\boxed{2}$

else go to $\boxed{5}$

5) $DIF_t = \sum_{p=j}^{t} d_p \ (p-j) - F(1)$

$DIF_{t-1} = F(1) - \sum_{p=j}^{t-1} d_p \ (p-j)$

6) If $DIF_t < DIF_{t-1}$ then, $X_j = \sum_{p=j}^{t} d_p$; j=t+1
t=t+1

else $X_j = \sum_{p=j}^{t-1} d_p$, j=t

go to $\boxed{2}$

7) End

2.5. SILVER AND MEAL HEURISTIC MODEL

The basic idea of the heuristic is based on the minimization of the total cost per unit time. It selects the order quantity in such a way that the total cost per unit time is minimized (9). Total cost per unit time can be expressed as:

$$TCUT(n) = \frac{1}{n} \left(S + h \sum_{t=1}^{n} d_t (t-1) \right) \qquad (2.12)$$

where TCUT(n) is total cost per unit time, n=1,2,3...etc. is the decision variable duration that replenishment quantity is to last.

The model of the heuristic evaluates TCUT(n) for increasing values of n until the following condition is satisfied:

$$TCUT(n+1) > TCUT(n) \qquad (2.13)$$

that is, until total cost per unit time starts to increase. When this happens, n is selected as the number of periods that the replenishment will cover. The stopping rule of the SM heuristic is obtained as follows:

From Eq.(2.12) and Eq.(2.13), we can write

$$\frac{1}{n+1} \left(S + h \sum_{t=1}^{n+1} d_t (t-1) \right) > \frac{1}{n} \left(S + h \sum_{t=1}^{n} d_t (t-1) \right)$$

By dividing both sides of above inequality by h and rearranging terms, we can obtain

$$\frac{\dfrac{S}{h} + \sum_{t=1}^{n+1} d_t(t-1)}{\dfrac{S}{h} + \sum_{t=1}^{n} d_t\,(t-1)} > \frac{n+1}{n}. \tag{2.14}$$

Since $F(n) = F(n-1) + (n-1)d_n$ and $F(1) = \dfrac{S}{h}$,

inequality Eq.(2.14) becomes

$$\frac{F(n+1)}{F(n)} > \frac{n+1}{n}$$

or

$$\frac{F(n) + nd_{n+1}}{F(n)} > \frac{n+1}{n} \tag{2.15}$$

By rearranging the terms of Eq.(2.15), we can obtain the stopping rule in the following form (10):

$$n^2\, d_{n+1} > F(n). \tag{2.16}$$

The application of this heuristic to the sample problem is given in Table 2.7 by using SM procedure.

Table 2.7. Silver and Meal Example

Period No	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Order Quantity	55				70	180	250	270	280			
Beginning Inventory	55	45	35	20	70	180	250	270	280	50	10	10
Ending Inventory	45	35	20	0	0	0	0	0	50	10	10	0

Ordering Cost = 6(TL 300.) =TL 1800.

Inventory carr. cost = 2(TL 170.) =TL 340.

Total cost = 1800 + 340 =TL 2140.

2.5.1. SM PROCEDURE

1) Let, j=1, t=1, $X_t = 0 \quad \forall t$, $F(1) = \frac{S}{h}$

2) Set k=t-j+1

$F(k) = F(k-1) + (k-1)d_t$ go to $\boxed{3}$

3) If t=T then, $X_j = \sum_{p=j}^{T} d_p$ go to $\boxed{8}$

else go to $\boxed{4}$

4) If $F(k) \geq k^2 d_{t+1}$ then, go to $\boxed{5}$

else go to $\boxed{6}$

5) If t=T or j=T then, $X_j = \sum_{p=j}^{T} d_p$ go to $\boxed{8}$

else t=t+1 go to $\boxed{2}$

6) Set $X_j = \sum_{p=j}^{t} d_p$, j=t+1, t=t+1 go to $\boxed{7}$

7) If t=T or j=T then, $X_j = \sum_{p=j}^{T} d_p$ go to $\boxed{8}$

else go to $\boxed{2}$

8) End.

2.6. GROFF MARGINAL COST MODEL

This heuristic is based on the marginal costs rather than the total costs. The traditional EOQ rule is established by increasing the lot-size as long as the marginal savings in the ordering cost is greater than the marginal cost increase in the inventory holding cost. Thus, the optimal lot-size is reached when the marginal increase in ordering cost equals to the marginal increase in inventory holding cost. By using the same analogy, this heuristic adds the future demands to the lot-size as long as the marginal increase in inventory holding cost for the period is less than the marginal decrease in ordering cost (11).

The marginal cost decrease for adding n+1st. period's demand to the lot is the decrease in ordering cost per period, or

$$\frac{S}{n} - \frac{S}{n+1} = \frac{S}{n(n+1)} \qquad (2.17)$$

Groff's model approximates the discrete inventory depletion to the uniform inventory depletion. Inventory holding cost for a horizon of n periods and n+1 periods can be determined as follows:

$$H(n) = \frac{1}{n} \left(\frac{1}{2} \; h.n \sum_{t=1}^{n} d_t \right) = \frac{1}{2} \; h \sum_{t=1}^{n} d_t$$

and

$$H(n+1)= \frac{1}{n+1}\left(\frac{1}{2}(n+1).h \sum_{t=1}^{n+1} d_t \right) = \frac{1}{2} h \sum_{t=1}^{n+1} d_t$$

then, the marginal cost increase from adding n+1st. period's demand to the lot is

$$H(n+1) - H(n) = \frac{1}{2} h \left(\sum_{t=1}^{n+1} d_t - \sum_{t=1}^{n} d_t \right) = \frac{1}{2} h\, d_{n+1} \qquad (2.18)$$

Therefore, the stopping rule of Groff's model will be

$$\frac{S}{(n+1)n} \leq \frac{1}{2} h\, d_{n+1} \qquad (2.19)$$

i.e., the marginal decrease in ordering cost is less than or equal to the marginal increase in inventory holding cost(4). The inequality (2.19) can be simplified and presented in the following form:

$$\frac{2S}{h} \leq n(n+1)d_{n+1} \qquad (2.20)$$

Application of the heuristic to the example problem is given in Table 2.8 by using GMC procedure.

Table 2.8. Groff Marginal Cost Model Example

Period No	1	2	3	4	5	6	7	8	9	10	11	12
Demand	10	10	15	20	70	180	250	270	230	40	0	10
Order Quantity	55				70	180	250	270	280			
Beginning Inventory	55	45	35	20	70	180	250	270	280	50	10	10
Ending Inventory	45	35	20	0	0	0	0	0	50	10	10	0

Ordering Cost = 6(TL 300.) =TL 1800.

Inv. carr. cost =TL 340.

Total cost =TL 2140.

2.6.1. GMC PROCEDURE

1) Let, $j=1$, $t=1$, $X_t=0$

2) If $t=T$ then, $X_j = \sum_{p=j}^{T} d_p$ go to $\boxed{7}$

   else go to $\boxed{3}$

3) Set, $k=t-j+1$

   $$f = \frac{1}{2} h\, d_{t+1}, \quad CST = \frac{S}{k(k+1)}$$

4) If $CST > f$ then, go to $\boxed{5}$

   else go to $\boxed{6}$

5) If $t=T$ or $j=T$ then, $X_j = \sum_{p=j}^{T} d_p$ go to $\boxed{7}$

   else $t=t+1$ go to $\boxed{2}$

6) Set, $X_j = \sum_{p=j}^{t} d_p$ and $j=t+1$ go to $\boxed{5}$

7) End.

CHAPTER III

3.1. COMPUTATIONAL DATA SETS FOR COMPARISON OF THE HEURISTIC MODELS

In general, there are two criteria to compare these heuristic models:(1) total cost of set-up and carrying inventory. (2) Computation time of heuristic model procedure. The second measure is a measure of the effort to find solutions. For the first criterion, the comparison is made in order to find the deviation of their total costs from the optimal solution. In other words, W/W model is used as a benchmark to measure the cost performance of the heuristic models. As the performances of the heuristic models are different under different data sets, it is hard to measure their performances exactly. The difficulty lies in the fact that the performances of the heuristic models vary, depending upon the variability of demand.

However, Kaimann (5) has prepared data sets which reduce this difficulty. These data sets which are given in Table 3.1 and 3.2 (in the Appendix A) have been prepared by using five different sets of cost data and seven different sets of demand data. As it can be seen from Figure 3.1., the demand data represents a variety of possible demand patterns that will encompass a wide range of the possible demand situations. Each of these demand patterns have the same total demand for the year, namely 1105 units.

There are 35 examples, all derived from the seven demand patterns and the five ordering costs. Totally 35 test problems are solved for each heuristic model.

The performances of all the heuristics are measured by using these data sets, and by comparing their results with those yielded by **WW**.

3.2. THE RESULT OF EXPERIMENTATION

Several heuristic models for determining the lot-size in single–item, deterministic and periodic review models have been outlined. These find extensive use in Material Requirements Planning systems.

Each of them starts with the current period and scans successive periods until the stopping rule is satisfied. Then an order is placed to satisfy the total requirements up to the stopping period, except for EOQ1 in which the order quantity is equal to EOQ. Then the same procedure is repeated for the remaining periods. Although the basic idea is the same, the stopping rules which characterize the heuristics are different. In this study, these stopping rules are also examined and results are summarized in Table 3.3. (in the Appendix A).

Almost all of the lot-sizing heuristics are based on the rationale underlying the EOQ, that is, they are developed by using one of the properties of the optimal solution of the EOQ. These properties may be summarized as follows:

- minimization of total costs per unit which is the objective of the LUC heuristic.

- minimization of total costs per unit time which is the objective of the SM heuristic.

- adding to inventory lot until the marginal increase in inventory holding costs is equated to the marginal decrease in ordering costs, which is the basic idea of GMC model.

- equating the total cost of ordering and the total inventory carrying cost which is the basic idea of PPB heuristic.

It is possible to conclude that all heuristics are developed based on the EOQ, each of them is approximated different approach to problems of cost minimization.

In this study, heuristic models for the single item, periodic review and deterministic model are compared and solutions are obtained in each case using the standard data sets available in the literature. In addition, all the heuristic models are analyzed with respect to structural properties and their stopping rules are summarized.

Moreover, an interactive package program which contains all the lot-sizing models are prepared. If the problem is solved by using computer, the user can use the interactive package program which is written in TURBO PASCAL language. The test problems are solved on a CORONA PC / XT-40 personal computer (in the Appendix B).

The results of 35 examples are given in Table 3.4.and Table 3.5.(Appendix A): The former exhibits the cost performance of the lot-sizing models, and the latter Table summarizes cost performance for each heuristic models.

As the Tables clearly indicate, since the WW model provides optimal results, it can be used as a yardstick to measure the comparative cost performance of the other lot sizing models.

The results indicate that the solution yielded by WW is best approximated by the by SM and by the GMC models. By using these heuristics, processing time can be decreased approximately twice. While the average percentage deviation of the results of SMH and GMC from the optimal solution is about .028%, the average of the said deviation for the other five heuristics are 1.523% .

3.2. DISCUSSION OF HEURISTIC MODELS WITH RESPECT TO THEIR PROCEDURE

According to the result of 35 test problems, SM model outperforms the other procedures. Also, near optimal solution is given by GMC model.

It is known that for these type of problems, WW model gives the optimal solution, the most important result of which can be expressed as below:

$$X_t * I_{t-1} = 0 \tag{3.1}$$

The expression (3.1) is stated that if the quantity of inventory carried to the next period is positive then the production at that period must be zero. Or, if the production quantity is greater than zero at any period then the carried inventory to this period must be zero. This is not the case in other discrete heuristic models. The SM heuristic is discrete lot-sizing model. For that reason, SM heuristic does not create remnant stock.

One of the features of EOQ1 and EOQ2 models is its consideration of total demand, which can be considered a weakness, because in lot-size determination problems the crucial variable is not total demand, but rather demand variation over the periods.

POQ has the similar property and weakness of considering total demand rather than demand variation over periods, but it determines an ordering interval. However, it makes a simple division without taking into consideration demand variation period by period as is the case in SM and GMC models, and therefore the former yields higher cost results as compared to the latter. And its ordering interval can cause the increase in total cost by increasing the order (production) cost or inventory carrying cost.

LUC, PPB, GMC and SM heuristics allow both ordering interval and order quantity to vary. Thus they have the capability of coping with the seasonal variability or lumpiness of the demand. Although the basic approach of them is same, SM heuristic has a better stopping rule: It considers the minimization of total costper unit time.

3.3. ANALYSIS OF RESULTS

The seven heuristics (i.e., EOQ1, EOQ2, POQ, LUC, PPB, SM and GMC) are tested by solving 35 test problems for each. As a result, totally 245 problems are solved by using an interactive package program which is written in Turbo Pascal Language and the results are depicted in Table 3.4. To facilitate a nalysis of these results, Table 3.6 is constructed. This Table presents the ratio of the total costs of set-up and inventory carrying found for each heuristic model for test problems when compared with WW algorithm.

These results are statistically analyzed via the one-way analysis of variance(ANOVA) technique. ANOVA method is used to examine if there are any significant differences between these heuristic models. Thus, the equivalence of the seven heuristics' total costs of set-up and inventory carrying means are set as a null hypothesis and the difference of one of them from the rest is set as an alternative hypothesis.

$H_o : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7$

H_A : At least one of the heuristics' total costs set-up and inventory carrying means differs from the rest.

The result of ANOVA output can be seen Table 3.7 in Appendix. As it can be observed from Table 3.7 the p-value[1] for corresponding to F-ratio = 36.282 and is approximately equal to zero. From the Table, we can reject or accept the null hypothesis either by looking at the F statistic or by looking at p-value. The observed F-test ratio is so big and p-value is so small that we can conclude that there is sufficient evidence to reject the null hypothesis. Thus, it can be concluded that means of the heuristics' total costs of set-up and inventory carrying significantly differ from each other.

It should also be concluded which heuristic models have shown better performances than the others. Then, the equivalence of the any two pair heuristics' total costs of set-up and inventory carrying them is set as an alternative hypothesis. That is,

$H_o : \mu_i = \mu_j$

$H_A : \mu_i \neq \mu_j$

The results of t-test computations for any two heuristic-model pairs are given on Table 3.8, results

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[1]The p-value for a test of hypothesis is the probability of obtaining a value of the test statistic as extreme or more extreme than the actual sample value when H_o is true.

indicate that there are statistically significant differences between SM, GMC on one hand,and the other five heuristic on the other[2]. It can be concluded that the performances of SM and GMC are better than other heuristics and a similar conclusion is indicated by confidence interval figures which are closer to 1(on Table 3.6.) for SM and GMC.

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[2]In doing these comparisons, the family-wise alpha is chosen 5% which implies that for each individual comparison, the attained p-value is compared with .000125 [=(.05/20)/2].

CHAPTER IV.

CONCLUSIONS

In this thesis, performances of heuristic models for single-item, deterministic, periodic-review lot-sizing problems are compared with the application to the Material Requirement Systems, since the production planning environments are generally affected by the decision to be made on MRP systems.

The mathematical programming formulation of lot-sizing problems are indicated and their assumptions are shown. Then the nature, meaning and importance of these assumptions and the implications of them are briefly discussed.

The method which yields an optimal solution for single-item, deterministic, periodic review and uncapacitated lot-sizing problems was developed by Wagner and Whitin in 1958, who developed a procedure that guarantees an optimal solution in terms of minimizing the total cost of replenishment and carrying inventory. However the procedure are received exteremely limited acceptance in practice, because of the relatively complex nature of its algorithm, the considerable computational effort required for its use and the possible need for a well-defined ending point for the demand pattern.

Instead, simpler heuristic models are resorted which result in reduced control costs that more than offset any extra replenishment or carrying costs that their use may incur. A scan is made of the relevant literature and seven of the heuristic models which yield best results are selected for a test of their performance: These are EOQ1, EOQ2, POQ, LUC, PPB, SM and GMC.

The basic concepts of each of these heuristic models are briefly summarized and a sample problem is solved for each.

Since the performances of heuristic models are different under different data sets, it is difficult to measure their performance exactly, because of the fact that the performances of the heuristic models vary depending upon the variability of demand. However, Kaimann (5) had prepared data sets(Table 3.1 and 3.2) which reduced this difficulty; and these data sets are used in our study for evaluating and comparing the performance of the heuristics selected.

A total of 35 test problems are made for each heuristic model, they are compared by using the standard Kaimann data sets. An interactive package programme written in TURBO PASCAL language which contains all the lot-sizing procedures are prepared. The results of the 35 examples (tests) are given in Table 3.4 and Table 3.5, which indicate that SM and GMC models outperform the others and best approximated the optimal solutions yielded by the WW.

According to the Table 3.7., for F-ratio=36.282 and p-value is approximately equal to zero. Thus, the observed F-test ratio is so big and p-value is so small. Thus, we can conclude that there is sufficient evidence to reject the null hypothesis. It can be concluded that the performance of the each heuristic differs from each other.

Having shown that the mean of the results obtained from 35 test cases are differed for each heuristic, t-tests are made for each pair of the seven heuristic models examined.

The results of t-tests for any two heuristic-model pairs (Table 3.8) indicate that there are significant differences between SM and GMC on the one hand, and the other five heuristics on the other.

When the mean of the results obtained from the former four heuristics and EOQ1 (expressed in terms of their ratio to the results obtained from WW algorithm) are compared to the analogue figures yielded by SM and GMC, extremely small p-values were obtained, the largest being about 0.00015.

Thus this study reconfirms that the performances of SM and GMC are better than those the other heuristics tested

Finally, for further research, a better solution model can be found by applying a more realistic stopping rule to the SM and GMC models, which have approximated the optimal results obtained from the WW model in our study.

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APPENDICES

APPENDIX A

TABLES

Table 3.1. Demand Data Sets

PERIOD	1	2	3	4	5	6	7
1	92	80	50	10	0	0	80
2	92	100	80	10	0	0	100
3	92	125	180	15	0	0	125
4	92	100	80	20	0	25	100
5	92	50	0	70	0	100	270
6	92	50	0	180	1105	300	50
7	92	100	180	250	0	400	230
8	92	125	150	270	0	250	0
9	92	125	10	230	0	30	50
10	92	100	100	40	0	0	0
11	92	50	180	0	0	0	0
12	93	100	95	10	0	0	60
TOTAL	1105	1105	1105	1105	1105	1105	1105
Standard Deviation	0.0	27.0	66.1	130	305	136	79.7

Table 3.2. Cost Data

	Ordering cost (TL)	Inventory carrying cost per unit per period (TL)
(a)	48	2
(b)	92	2
(c)	120	2
(d)	206	2
(e)	300	2

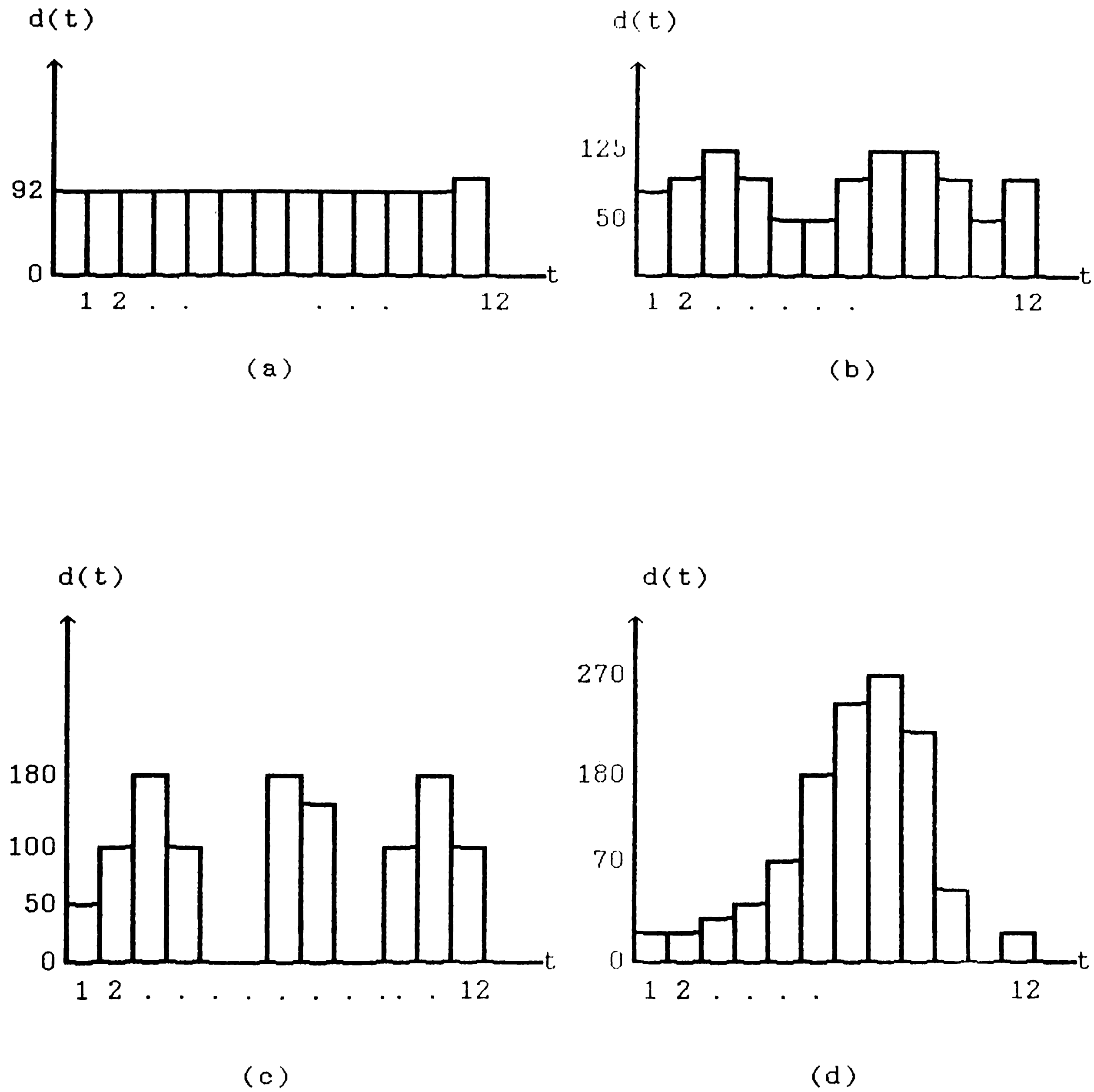


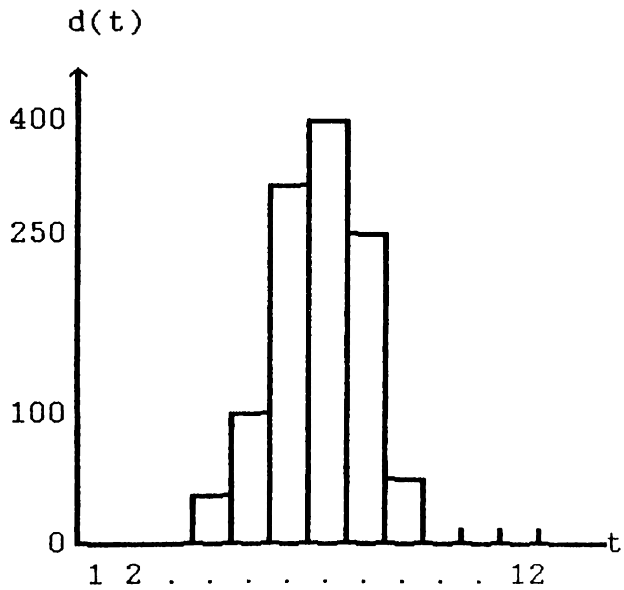
Figure 3.1. Schematic Representation of the Demand Patterns:

(a) Demand data one (e) Demand data two

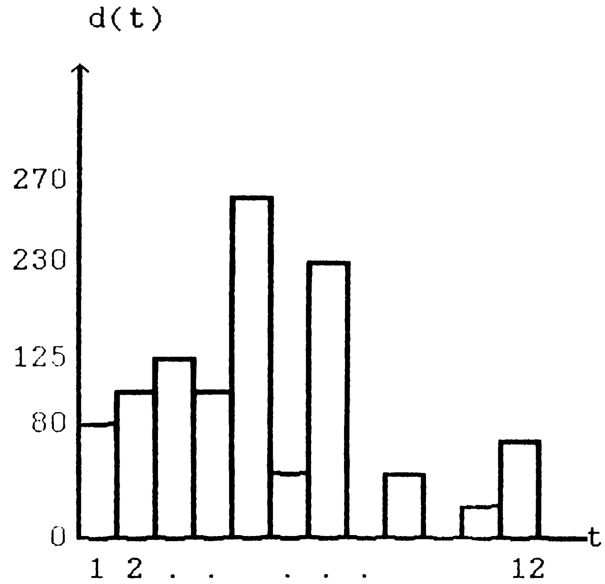
(b) Demand data three (f) Demand data four

(c) Demand data six (g) Demand data seven

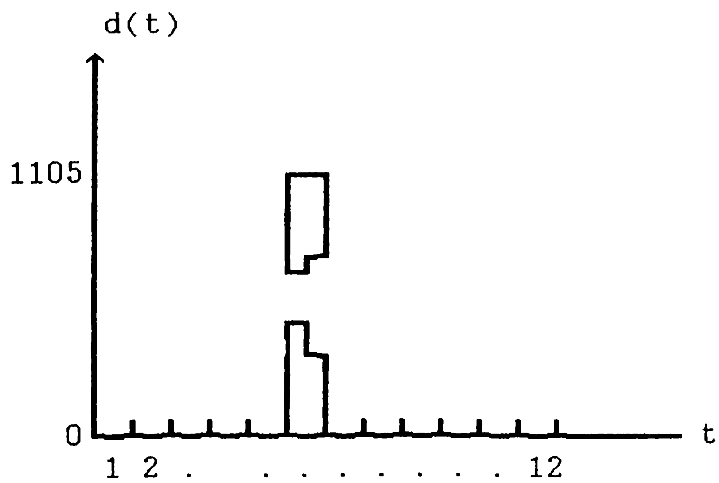
(d) Demand data five



(e)



(f)



(g)

Figure 3.1(cont´d)

Table 3.3. Summary of the Stopping Rules of the Heuristics

Name of Model	Stopping Rule
EOQ1	$I_{t-1} < d_t$
EOQ2	$EOQ < \sum_{t=1}^{k} d_t$
POQ	$n > EOQ$
LUC	$n \sum_{t=1}^{T} d_t > F(n)$
PPB	$\frac{2S}{h} < F(n)$
SM	$n^2 d_{n+1} > F(n)$
GMC	$\frac{2S}{h} \leq n(n+1)d_{n+1}$

TABLE 3.4.

COST PERFORMANCE OF THE LOT-SIZING MODELS

		PROBLEM TYPE						
Name of Model	I	test1a	test2a	test3a	test4a	test5a	test6a	test7a
WW	TC	576	576	452	484	48	288	480
	t	0.07	0.06	0.06	0.06	0.03	0.04	0.06
EOQ1	TC	576	704	672	774	48	672	756
	t	0.02	0.01	0.02	0.02	0.01	0.02	0.02
EOQ2	TC	576	576	632	576	48	288	480
	t	0.02	0.02	0.02	0.02	0.01	0.02	0.02
POQ	TC	576	576	480	528	48	288	480
	t	0.02	0.02	0.02	0.02	0.02	0.01	0.01
LUC	TC	576	576	774	492	48	288	480
	t	0.03	0.02	0.02	0.02	0.02	0.02	0.02
PPB	TC	576	576	452	524	48	300	480
	t	0.03	0.03	0.03	0.03	0.02	0.03	0.02
SM	TC	576	576	452	492	48	288	480
	t	0.02	0.02	0.03	0.02	0.02	0.02	0.03
GMC	TC	576	576	452	492	48	288	480
	t	0.02	0.02	0.03	0.02	0.02	0.03	0.02

I: Total Cost (TC) and time (t); TC(TL) and t(sec)

TABLE 3.4.(cont'd)

Name of Model	I	PROBLEM TYPE						
		test1b	test2b	test3b	test4b	test5b	test6b	test7b
WW	TC	1104	1104	848	836	92	520	920
	t	0.06	0.06	0.06	0.06	0.03	0.04	0.06
EOQ1	TC	1104	1810	1496	1550	92	1300	1184
	t	0.02	0.01	0.02	0.02	0.01	0.02	0.02
EOQ2	TC	1104	1112	1096	1352	92	660	988
	t	0.02	0.02	0.02	0.02	0.01	0.02	0.02
POQ	TC	1104	1104	920	1012	92	552	920
	t	0.02	0.01	0.01	0.02	0.01	0.02	0.02
LUC	TC	1104	1104	1126	956	92	660	948
	t	0.02	0.02	0.02	0.03	0.02	0.02	0.02
PPB	TC	1104	1120	984	904	92	552	996
	t	0.03	0.03	0.03	0.03	0.02	0.03	0.03
SM	TC	1104	1104	848	876	92	520	920
	t	0.02	0.03	0.02	0.02	0.02	0.03	0.02
GMC	TC	1104	1104	848	896	92	520	920
	t	0.02	0.02	0.02	0.03	0.02	0.02	0.02

TABLE 3.4. (cont'd)

Name of Model	I	PROBLEM TYPE						
		test1c	test2c	test3c	test4c	test5c	test6c	test7c
WW	TC	1440	1440	1100	1040	120	660	1180
	t	0.06	0.06	0.06	0.06	0.02	0.04	0.06
EOQ1	TC	2506	1980	2170	2000	120	1650	1720
	t	0.02	0.02	0.02	0.02	0.02	0.02	0.02
EOQ2	TC	1440	1500	1320	1520	120	800	1240
	t	0.02	0.02	0.02	0.02	0.02	0.02	0.02
POQ	TC	1440	1440	1200	1320	120	720	1200
	t	0.01	0.01	0.02	0.02	0.01	0.02	0.02
LUC	TC	1440	1500	1390	1180	120	800	1540
	t	0.02	0.02	0.02	0.02	0.02	0.02	0.02
PPB	TC	1826	1740	1250	1100	120	800	1420
	t	0.02	0.03	0.03	0.02	0.02	0.02	0.03
SM	TC	1440	1440	1100	1060	120	660	1180
	t	0.03	0.03	0.02	0.02	0.02	0.02	0.02
GMC	TC	1440	1440	1100	1120	120	660	1180
	t	0.02	0.02	0.02	0.02	0.02	0.03	0.02

TABLE 3.4.(cont'd)

Name of Model	I	PROBLEM TYPE						
		test1d	test2d	test3d	test4d	test5d	test6d	test7d
WW	TC	2342	2248	1766	1576	206	1084	1850
	t	0.06	0.06	0.06	0.06	0.02	0.04	0.06
EOQ1	TC	3232	3768	2642	2694	206	2146	3068
	t	0.01	0.02	0.02	0.01	0.02	0.02	0.02
EOQ2	TC	2362	2248	2008	2036	206	1230	2162
	t	0.02	0.02	0.02	0.02	0.02	0.02	0.02
POQ	TC	2472	2472	2060	2266	206	1236	2060
	t	0.02	0.02	0.01	0.01	0.01	0.01	0.01
LUC	TC	2342	2248	1992	1836	206	1230	2596
	t	0.02	0.03	0.02	0.02	0.02	0.02	0.02
PPB	TC	2342	2386	2040	1836	206	1230	1850
	t	0.03	0.03	0.02	0.02	0.02	0.03	0.02
SM	TC	2342	2436	1766	1596	206	1084	1856
	t	0.02	0.02	0.03	0.02	0.02	0.03	0.03
GMC	TC	2342	2342	1766	1596	206	1084	1856
	t	0.03	0.02	0.02	0.02	0.02	0.02	0.02

TABLE 3.4.(cont'd)

		PROBLEM TYPE						
Name of Model	I	test1e	test2e	test3e	test4e	test5e	test6e	test7e
WW	TC	2906	2950	2330	2140	300	1460	2320
	t	0.06	0.06	0.06	0.06	0.02	0.04	0.06
EOQ1	TC	4014	4330	3846	3760	300	2952	3656
	t	0.02	0.02	0.01	0.01	0.02	0.01	0.02
EOQ2	TC	2906	3000	2580	2600	300	1700	3280
	t	0.02	0.02	0.02	0.01	0.02	0.02	0.02
POQ	TC	2906	2950	2860	2840	300	1960	2420
	t	0.02	0.01	0.02	0.02	0.01	0.02	0.01
LUC	TC	2906	3050	2640	2600	300	2600	2820
	t	0.03	0.02	0.03	0.03	0.02	0.02	0.02
PPB	TC	2906	2950	2510	2640	300	1900	2320
	t	0.03	0.02	0.03	0.03	0.02	0.03	0.02
SM	TC	2906	2950	2350	2140	300	1460	2420
	t	0.02	0.02	0.02	0.02	0.02	0.02	0.02
GMC	TC	2906	2950	2330	2140	300	1460	2460
	t	0.03	0.02	0.03	0.02	0.02	0.02	0.02

TABLE 3.5.

SUMMARY OF COST PERFORMANCE OF THE LOT-SIZING MODELS

Lotsizing models	average obj. func. value (TL)	average percentage cost increase over W/W algorithm(%)	average total process time (second)	number of times a lot sizing model finds the optimum
WW	1164.17	0.000	.052	35
EOQ1	1871.37	1.734	.017	7
EOQ2	1318.22	.378	.019	13
POQ	1289.37	.307	.015	16
LUC	1330.28	.406	.021	15
PPB	1268.00	.255	.026	15
SM	1175.65	.028	.022	27
GMC	1175.82	.029	.022	28

TABLE 3.6.

	EOQ1/WW	EOQ2/WW	POQ/WW	LUC/WW	PPB/WW	SM/WW	GMC/WW
Test1a	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test2a	1.222	1.000	1.000	1.000	1.000	1.000	1.000
Test3a	1.486	1.398	1.061	1.712	1.000	1.000	1.000
Test4a	1.599	1.190	1.090	1.016	1.088	1.016	1.016
Test5a	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test6a	2.333	1.000	1.000	1.000	1.041	1.000	1.000
Test7a	1.575	1.000	1.000	1.000	1.000	1.000	1.000
Test1b	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test2b	1.639	1.007	1.000	1.000	1.014	1.000	1.000
Test3b	1.764	1.292	1.084	1.327	1.160	1.000	1.000
Test4b	1.854	1.617	1.210	1.143	1.081	1.047	1.071
Test5b	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test6b	2.500	1.269	1.061	1.269	1.061	1.000	1.000
Test7b	1.286	1.073	1.000	1.030	1.082	1.000	1.000
Test1c	1.740	1.000	1.000	1.000	1.268	1.000	1.000
Test2c	1.375	1.041	1.000	1.041	1.208	1.000	1.000
Test3c	1.972	1.200	1.090	1.263	1.136	1.000	1.000
Test4c	1.923	1.461	1.269	1.134	1.057	1.019	1.076
Test5c	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test6c	2.500	1.212	1.090	1.212	1.212	1.000	1.000
Test7c	1.457	1.050	1.016	1.305	1.203	1.000	1.000
Test1d	1.380	1.008	1.055	1.000	1.000	1.000	1.000
Test2d	1.676	1.000	1.099	1.000	1.061	1.083	1.041
Test3d	1.496	1.137	1.166	1.127	1.155	1.000	1.000

TABLE 3.6.(cont'd)

Test4d	1.709	1.291	1.437	1.164	1.164	1.012	1.012
Test5d	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test6d	1.979	1.134	1.140	1.134	1.134	1.000	1.000
Test7d	1.658	1.168	1.113	1.403	1.000	1.003	1.003
Test1e	1.381	1.000	1.000	1.000	1.000	1.000	1.000
Test2e	1.467	1.016	1.000	1.033	1.000	1.000	1.000
Test3e	1.650	1.107	1.227	1.133	1.077	1.008	1.000
Test4e	1.757	1.214	1.327	1.214	1.233	1.000	1.000
Test5e	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Test6e	2.021	1.164	1.342	1.780	1.301	1.000	1.000
Test7e	1.575	1.413	1.043	1.215	1.000	1.043	1.060
Average	1.570	1.127	1.083	1.133	1.078	1.007	1.008
Min	1.222	1.007	1.016	1.016	1.014	1.003	1.003
Max	2.500	1.617	1.437	1.780	1.301	1.047	1.076
Std(σ_i)	.4138	.1605	.1154	.1924	.0915	.0174	.0205

	99% Confidence Interval	95% Confidence Interval
EOQ1/WW	(1.3900, 1.7500)	(1.4330, 1.7000)
EOQ2/WW	(1.0570, 1.1960)	(1.0740, 1.1800)
POQ/WW	(1.0320, 1.1330)	(1.0447 , 1.1212)
LUC/WW	(1.0491 , 1.2168)	(1.0693 , 1.1967)
PPB/WW	(1.0381 , 1.1178)	(1.0476 , 1.1083)
SM/WW	(1.0070, 1.0130)	(1.0010, 1.0110)
GMC/WW	(1.0070, 1.0160)	(1.0020, 1.0140)

TABLE 3.7.

ANALYSIS OF VARIANCE

ONE-WAY ANOVA

GROUP	MEAN	N
EOQ1	1.571	35
EOQ2	1.127	35
POQ	1.083	35
LUC	1.033	35
PPB	1.078	35
SM	1.007	35
GMC	1.008	35
GRAND MEAN	1.144	245

VARIABLE 1:COST

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE	F RATIO	PROB.
BETWEEN	7.975	6	1.329	36.282	.000E-00
WITHIN	8.719	238	.037		
TOTAL	16.694	244			

TABLE 3.8.

The Comparison of Difference Between any Two Group Means

two heu. method		t-test statistic	D.F.	p-values	conclusion
EOQ1	POQ	6.701	68	2.35E-09	There are significant differences btw EOQ1 and other heu.
	LUC	5.674	"	1.56E-07	
	PPB	6.877	"	1.18E-09	
	SM	8.057	"	8.66E-12	
	GMC	8.034	"	9.47E-12	
EOQ2	POQ	1.312	"	.05	No diff.
	LUC	-.130	"	.44	"
	PPB	1.585	"	.05	"
	SM	4.431	"	1.75E-05	Signif. diff.
	GMC	4.370	"	2.17E-05	"
POQ	LUC	-1.307	"	.09	No diff.
	PPB	.218	"	.41	"
	SM	3.894	"	1.13E-04	Signif. diff.
	GMC	3.808	"	1.51E-04	"
LUC	PPB	1.527	"	.06	No diff.
	SM	3.871	"	1.22E-04	Signif. diff.
	GMC	3.823	"	1.44E-04	"
PPB	SM	4.535	"	1.20E-05	Signif. diff.
	GMC	4.418	"	1.83-E05	"
SM	GMC	-.302	"	.38	No diff.

APPENDIX B

LISTING OF THE COMPUTER

VARIABLES DESCRIPTION

Dem.....................The array of demands for any period.

d .... " " " cumulative demands.

I .... " " " inventory costs.

apk .... " " " set-up costs.

X .... " " " production quantities.

HD .... " " " total demands among any periods.

Y .... " " " total production at each node.

R .... " " " cumulative production.

f .... " " " total cost

FC...................... " " " cost at each node

---

infl and **oufl** are text files

inf and **ouf** are strings and indicates name of files.

integer variables:

per................. number of periods.

dwrk, ss, lot....... indices for some summations.

t, j, k, p .... indices for time periods.

real variables:

sec, sec1, 2, 3, 4...represents time

ave.................." average of any dimensions considered.

PROCEDURES DESCRIPTION

Procedure readchoice .... used to write and select the options which are seven heuristic models and **WW** algorithm.

Procedure EOQ1.................... used to calculate order (or production)quantities for EOQ1 model and the total cost. In this procedure, EOQLOT finds the order quantities.

Procedure EOQ2.................... used to find production quantities for EOQ2 model and the total cost. In this procedure, EOQLOT finds order quantities.

Procedure GRMC .... used to find production (or purchase) quantities which is based on the marginal costs and also to calculate total cost.

Procedure LUC .... used to find production (or purchase) quantities which is based on the minimization of the unit cost and to calculate total cost.

Procedure SMH .... used to find production (or purchase) quantities in considered periods which is based on the minimization of the total cost per unit time and also to calculate total cost.

Procedure WW .... used to find production (or purchase) quantities which is based on WW algorithm and also to calculate total cost.

Procedure Getdata .... used to read datas from input file.

Fuction.......................... used to find time in occuring each seven heu. procedure and **WW** algorithm

LIST OF PROGRAM

```
dmt;
uses dos;
const
 big=1.0E9;
type
 i1dim=array [0..365] of integer;
 r1dim=array [0..365] of real;
    smallint=1..9;
var
 x,dem,d,I,xx,apk,HD,R,Y:i1dim;
 f,FC:r1dim;
 infl,oufl:text;
 inf,ouf:string[20];
 t,per,j,k,dwrk,lot,p,ss:integer;
 ave,cost,s,h,cst,sec,sec1,sec2,sec3,sec4:real;
              choice:smallint;
procedure READCHOICE(var CHOICE:SMALLINT);
var
 TEMPCHOICE:INTEGER;
procedure PRINTMENU;
begin
 WRITELN('**************CHOOSE AN OPTION**************************');
 WRITELN('**************************************************************');
 WRITELN('  1)Wagner Whitin                                              ');
 WRITELN;
 WRITELN('  2)Economic order quantity one(EOQ1)       ');
 WRITELN;
 WRITELN('  3)Economic order quantity two(EOQ2)');
 WRITELN;
 WRITELN('  4)Period order quantity(POQ)');
 WRITELN;
 WRITELN('  5)Least unit cost(LUC)');
 WRITELN;
 WRITELN('  6)Part period balancing(PPB)  ');
 WRITELN;
 WRITELN('  7)Silver and Meal heuristic(SMH)  ');
 WRITELN;
 WRITELN('  8)Groff marginal cost algorithm(GRMC)');
 writeln;
 WRITELN('  9)EXIT');
end;
begin
 PRINTMENU;
 repeat
  WRITE('Select an option(1 through 9):');
  READLN(TEMPCHOICE)
 until TEMPCHOICE in [1..9];
 CHOICE:=TEMPCHOICE
end;

procedure EOQLOT;
begin
dwrk:=0; d[0]:=0; dem[0]:=0; I[0]:=0;x[0]:=0;
for t:=1 to per do
 begin
 dwrk:=dwrk+dem[t];
 d[t]:=dwrk;
 end;
 ave:=d[per]/per;
 lot:=round(sqrt((2*s*ave)/h));
```

```
end;

procedure EOQ2;
var
 min1,min2,ss:integer;
begin
 EOQLOT;
 p:=1; for t:=1 to per do begin x[t]:=0;  apk[t]:=0; end;
 repeat
 k:=p;
    ss:=d[per]-d[p-1];
 if ss<lot then
                        begin x[p]:=ss;k:=per+1; end
else
BEGIN
 if (dem[p]>lot) or (dem[p]=0)   then begin   x[p]:=dem[p]; p:=p+1;  end
 else
  begin
    apk[k]:=d[k]-d[p-1];
   while (apk[k]<=lot) and (k<=per) do
   begin
    k:=k+1;
    if k<=per then apk[k]:=d[k]-d[p-1];
   end;

  apk[k-1]:=d[k-1]-d[p-1];
  min1:=apk[k]-lot;
  min2:=apk[k-1]-lot;   if min2<0 then min2:=-min2;
  if min1<=min2 then begin x[p]:=  d[k]-d[p-1]; p:=k+1;    end
  else              begin x[p]:=d[k-1]-d[p-1]; p:=k; end;

 end

END
until k>per;

 cost:=0; xx[0]:=0;
 for t:=1 to per do
  begin
  xx[t]:=xx[t-1]+x[t];
  if x[t]>0 then cost:=cost+s;
  ss:=xx[t]-d[t];
  cost:=cost+(ss)*h;
  end;

 writeln(oufl,'EOQ2 RESULT');
 writeln(oufl,'-----------');
 for t:=1 to per do write(oufl,x[t]:5);
 writeln(oufl,'      cost=',cost:6:2);

end;


procedure  GETDATA;
begin
 write('input file=');readln(inf);
 assign(infl,inf); reset(infl);
 write('output file=');readln(ouf);
 assign(oufl,ouf); {rewrite(oufl);}
 readln(infl,per);
```

```
 for t:=1 to per do
  read(infl,dem[t]);
 readln(infl);
 readln(infl,s,h);
end;

procedure GRMC;
var f:real;
begin
 dwrk:=0; d[0]:=0; dem[0]:=0; x[0]:=0;
for t:=1 to per do
 begin
 dwrk:=dwrk+dem[t];
 d[t]:=dwrk; x[t]:=0;
 end;

  j:=1; t:=1;
  repeat
   k:=t-j+1;
   f:=1/2*h*dem[t+1];
   cst:=s/(k*(k+1));
   if cst<=f then
    begin
     dwrk:=0;
     for p:=j to t do dwrk:=dwrk+dem[p];
     x[j]:=dwrk;
     j:=t+1;
    end;
   t:=t+1;
  until (t=per) ;

 if (f<CST) or (p<per) then
  begin
  dwrk:=0;
  for p:=j to per do dwrk:=dwrk+dem[p];
  x[j]:=dwrk;
  end;

 cost:=0; xx[0]:=0;
 for t:=1 to per do
  begin
  xx[t]:=xx[t-1]+x[t];
  if x[t]>0 then cost:=cost+s;
  ss:=xx[t]-d[t];
  cost:=cost+(ss)*h;
  end;

 writeln(oufl,'GRMC RESULT');
 writeln(oufl,'-----------');
 for t:=1 to per do write(oufl,x[t]:5);
 writeln(oufl,'      cost=',cost:8:2);
end;




procedure PPB;
var dift,dift1:real;
z:integer;
```

```
begin
dwrk:=0; d[0]:=0; dem[0]:=0; x[0]:=0;
for t:=1 to per do
 begin
 dwrk:=dwrk+dem[t];
 d[t]:=dwrk; x[t]:=0;
 end;
 z:=0;
 repeat
 z:=z+1;
 until dem[z]>0;


 j:=z; t:=z; f[1]:=s/h; cst:=2*s/h;
 repeat
  t:=t+1;
  k:=t-j+1;
  if (dem[t]=0) and (t=per-1) then begin
             t:=t+1;if t=per then j:=j+1; k:=t-j+1; end;
  f[k]:=f[k-1]+((k-1)*dem[t]);
  if f[k]>CST then
   begin
    dwrk:=0;
    for p:=t downto j do dwrk:=dwrk+dem[p]*(p-j);
    dift:=dwrk-(s/h);

    dwrk:=0;
    for p:=t-1 downto j do dwrk:=dwrk+dem[p]*(p-j);
    dift1:=(s/h)-dwrk;

    if dift<dift1 then
     begin
     dwrk:=0;
     for p:=j to t do dwrk:=dwrk+dem[p];
     x[j]:=dwrk;  j:=t+1;
     end
    else
     begin
     dwrk:=0;
     for p:=j to t-1 do dwrk:=dwrk+dem[p];
     x[j]:=dwrk;  j:=t;
     end;
   end
   else x[j]:=dem[j];
{  end}
until (t=per) or (j=per);

if (f[k]<=CST) or (p<per) then
 begin
 dwrk:=0;
 for p:=j to per do dwrk:=dwrk+dem[p];
 x[j]:=dwrk;
 end;

cost:=0; xx[0]:=0;
for t:=1 to per do
 begin
 xx[t]:=xx[t-1]+x[t];
 if x[t]>0 then cost:=cost+s;
 ss:=xx[t]-d[t];
```

```
  cost:=cost+(ss)*h;
  end;

 writeln(oufl,'PPB RESULT');
 writeln(oufl,'-----------');
 for t:=1 to per do write(oufl,x[t]:5);
 writeln(oufl,'      cost=',cost:8:2);
end;



procedure SMH;
begin
 dwrk:=0; d[0]:=0; dem[0]:=0; x[0]:=0;
for t:=1 to per do
 begin
 dwrk:=dwrk+dem[t];
 d[t]:=dwrk; x[t]:=0;
 end;

 j:=1; t:=1; f[1]:=s/h;
 repeat
  k:=t-j+1;
  if k=1 then f[1]:=s/h
  else f[k]:=f[k-1]+((k-1)*dem[t]);
  cst:=k*k*dem[t+1];
  if cst>f[k] then
   begin
    dwrk:=0;
    for p:=j to t do dwrk:=dwrk+dem[p];
    x[j]:=dwrk;
    j:=t+1;    t:=t+1;
   end
  else t:=t+1
 until (j=per) or (t=per);

  if (f[k]>=cst) or (p<per) then
  begin
  dwrk:=0;
  for p:=j to per do dwrk:=dwrk+dem[p];
  x[j]:=dwrk;
  end;

 cost:=0; xx[0]:=0;
  for t:=1 to per do
   begin
   xx[t]:=xx[t-1]+x[t];
   if x[t]>0 then cost:=cost+s;
   ss:=xx[t]-d[t];
   cost:=cost+(ss)*h;
   end;

  writeln(oufl,'SMH RESULT');
  writeln(oufl,'-----------');
  for t:=1 to per do write(oufl,x[t]:5);
  writeln(oufl,'      cost=',cost:8:2);

 end;
```

```
procedure www;
var xx:real;
     l,z:integer;
BEGIN
         dwrk:=0; d[0]:=0;      HD[0]:=0;
         for t:=1 to per do
         begin
              dwrk:=dwrk+dem[t];
              d[t]:=dwrk;   r[t]:=0;
              HD[T]:=HD[T-1]+d[t];
         end;
  for J:=0 to per do
   R[J]:=0;
 t:=per+1;
 repeat
  t:=t-1;
 until dem[t]>0;

   FC[0]:=0;
   for K:=1 to t do
    begin
     IF DEM[K]=0 THEN BEGIN  FC[K]:=FC[K-1]; Y[K]:=K-1; END
     ELSE BEGIN
     FC[K]:=big; {if dem[k]>0 then l:=1 else l:=0;}
     for J:=0 to K-1 do
      begin
       XX:=FC[J]+s{*l)}+h*(d[K]*(K-1-J))-h*(HD[K-1]-HD[J]);
       if XX<FC[K] then
        BEGIN FC[K]:=XX; Y[K]:=J; END;
      end;
              END
    end;
   cost:=FC[t];

 { K:=per;
 Y[0]:=-1;
  repeat
     R[Y[K]+1]:=d[K]-d[Y[K]];
     K:=Y[K];
  until K=-1;}

   k:=t;
   repeat
    l:=y[k];
    r[l+1]:=d[k]-d[l];
    for j:=l+2 to k do r[j]:=0;
    k:=l;
   until l=0;
```

```
 writeln(oufl,'WWC RESULT');
 writeln(oufl,'-----------');
 for t:=1 to per do write(oufl,R[t]:5);
 writeln(oufl,'      cost=',cost:8:2);

END;

procedure EOQ1;
begin
 EOQLOT;
 cost:=0; xx[0]:=0;
 for t:=1 to per do
  begin
   if (I[t-1]>=dem[t]) then x[t]:=0
   else
    begin
     dwrk:=dem[t]-I[t-1];
     if dwrk>lot then x[t]:=dwrk
     else x[t]:=lot;
     cost:=cost+s;
    end;
   I[t]:=I[t-1]+x[t]-dem[t];  cost:=cost+(h*I[t]);
  end;{t}
 writeln(oufl,'EOQ1 RESULT');
 writeln(oufl,'-----------');
 for t:=1 to per do write(oufl,x[t]:5);
 writeln(oufl,'      cost=',cost:8:2);
end;

procedure POQ;
var EOI,kt,z:integer;
begin
 eoqlot;
 dwrk:=0; d[0]:=0;      HD[0]:=0;
        for t:=1 to per do
        begin
             dwrk:=dwrk+dem[t];
             d[t]:=dwrk; x[t]:=0;
        end;

z:=0; repeat z:=z+1; until dem[z]>0;
 EOI:=round(per/(d[per]/LOT));
 t:=z;
 repeat
  if dem[t]=0 then t:=t+1;
  kt:=EOI+t-1;
  if per<kt then kt:=per;
  x[t]:=d[kt]-d[t-1];
  t:=kt+1
 until kt>=per;

 cost:=0; xx[0]:=0;
 for t:=1 to per do
  begin
  xx[t]:=xx[t-1]+x[t];
  if x[t]>0 then cost:=cost+s;
  ss:=xx[t]-d[t];
  cost:=cost+(ss)*h;
  end;
```

```
 writeln(oufl,'POQ RESULT');
 writeln(oufl,'-----------');
 for t:=1 to per do write(oufl,x[t]:5);
 writeln(oufl,'      cost=',cost:8:2);

end;


procedure LUC;
var cost1,n:integer;
begin
 dwrk:=0; d[0]:=0; dem[0]:=0; f[0]:=0;x[0]:=0;
for t:=1 to per do
 begin
 dwrk:=dwrk+dem[t];
 d[t]:=dwrk;
 end;

 for t:=1 to t do x[t]:=0;           k:=0;
 p:=1; f[0]:=0;

 repeat
  n:=1; f[1]:=s/h;
IF DEM[P]=0 THEN P:=P+1
ELSE BEGIN

  if p=per then x[per]:=dem[per]
  else
   begin
    if dem[p]>f[1] then begin x[p]:=dem[p]; p:=p+1;
        if p+n-1=per then begin dwrk:=0; for k:=p to per do dwrk:=dwrk+dem[k];
                              x[p]:=dwrk; end;
                end
    else
     begin
      repeat
      n:=n+1;
      cost:=0; for k:=p to p+n-1 do cost:=cost+dem[k];
      cost:=cost*n;
      f[n]:=f[n-1]+(n-1)*dem[p+n-1];
      until (cost>f[n]) or (p+n-1=per);
      if p+n-1=per then begin dwrk:=0; for k:=p to per do dwrk:=dwrk+dem[k];
                              x[p]:=dwrk; end;
      if cost>f[n] then begin dwrk:=0; for k:=p to p+n-1 do dwrk:=dwrk+dem[k];
                              x[p]:=dwrk; p:=p+n; end;

     end
   end
 END;
if P+N-1=per then begin x[p]:=d[per]-d[p-1]; k:=1; end;
  if p=per then x[p]:=dem[per];
 N:=1;
until (p>=per) or (k=1){(p+n-1=per)};

 cost:=0; xx[0]:=0;
 for t:=1 to per do
  begin
  xx[t]:=xx[t-1]+x[t];
  if x[t]>0 then cost:=cost+s;
  ss:=xx[t]-d[t];
```

```
    cost:=cost+(ss)*h;
    end;

   writeln(oufl,'LUC RESULT');
   writeln(oufl,'-----------');
   for t:=1 to per do write(oufl,x[t]:5);
   writeln(oufl,'      cost=',cost:8:2);
  end;
  function SECONDS:REAL;
  type
      RegList = record
        case integer of
         0 : (AX, BX, CX, DX, BP, SI, DI, DS, ES, FLAGS: word);
         1 : (AL, AH, BL, BH, CL, CH, DL, DH: byte)
      end;
  var
     Reg : Registers;
     Hr, Min, Sec, Hun:byte;
  begin
       Reg.AX:= $2C00;
       Msdos(Reg);
       Hr := hi(Reg.CX); Min := lo(Reg.CX);
       Sec:= hi(Reg.DX); Hun:= lo(Reg.DX);
       SECONDS:= 3600.0*Hr+60.0*Min+Sec+0.01*Hun
  end; {SECONDS}



 begin
  for k:=1 to 25 do writeln;
  GETDATA;       for k:=1 to 15 do writeln;
 repeat
  readchoice(choice);
 case choice of
 1:begin  rewrite(oufl);
 {*********************************}
        sec:=seconds;
  WWW;
        sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec');
                                           close(oufl);         end;
 2:begin  rewrite(oufl);
 {*********************************}
        sec:=seconds;
  EOQ1;
        sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec'
                                           close(oufl)
 3:begin  rewrite(oufl);
 {*********************************}
        sec:=seconds;
  EOQ2;
        sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec');
                                           close(oufl)      end;
 4:begin  rewrite(oufl);
 {*********************************}
        sec:=seconds;
 POQ;
        sec1:=(seconds-sec); writeln(oufl,'time=':71,sec1:3:2,' sec');
                                           close(oufl)      end;
 5:begin  rewrite(oufl);
 {*********************************}
```

```
       sec:=seconds;
 LUC;
       sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec'
                                                close(oufl)      end;
6:begin   rewrite(oufl);
{*********************************}
       sec:=seconds;
 PPB;
       sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec');
                                                close(oufl)      end;
7:begin   rewrite(oufl);
{*********************************}
       sec:=seconds;
 SMH;
       sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec');
                                      close(oufl)      end;
8:begin   rewrite(oufl);
{*********************************}
       sec:=seconds;
 GRMC;
       sec1:=seconds-sec; writeln(oufl,'time=':71,sec1:3:2,' sec');
                close(oufl)      end;

{*********************************}
 9: writeln('**** SEE YOU LATER ********')
end
until choice=9;

{close(oufl);}
end.
```